Department of Electrical and Electronic Engineering,
Bangladesh University of Engineering and Technology

EEE 303: Digital Electronics

Level 3/ Term 1/ Section A

February 2013 Session

Course Teacher

Upal Mahbub
Assistant Professor,
Dept. of EEE, BUET.
E-mail:
omeecd@eee.buet.ac.bd;
omeecd@yahoo.com

Topic of the Day

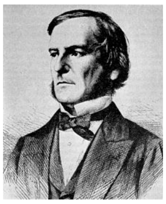
X + 1 = 1 !?

The BOOLEAN Logic

LECTURE 2

BOOLEAN Algebra: The History

1847: English mathematician George Boole introduced BOOLEAN algebra in his book, 'The Mathematical Analysis of Logic'. According to Huntington the term "Boolean algebra" was first suggested by Sheffer in 1913.



George Boole
(1815 - 1864)

BOOLEAN Algebra: The History

1937: Shannon is credited with founding both digital computer and digital circuit design theory, when, as a 21-year-old master's degree student at MIT, he wrote his thesis demonstrating that electrical applications of BOOLEAN algebra could construct and resolve any logical, numerical relationship.

It has been claimed that this was the most important master's thesis of all time.

Claude Elwood Shannon
(1916 –2001)

The Axioms

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	Identity element
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	Idempotence
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	Complement
9.	$\overline{\overline{X}} = X$			Involution
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$(X + Y) + Z = X + (Y + Z)$	13.	$(XY) Z = X(Y Z)$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y) (X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan 's
18	$X + X.Y = X$	19	$X.(X+Y) = X$	Absorption

Boolean Algebraic Proof – Example 1

- **A + A · B = A** **(Absorption Theorem)**

Proof Steps	**Justification**
A + A · B	
= A · 1 + A · B	**Identity element: A · 1 = A**
= A · (1 + B)	**Distributive**
= A · 1	**1 + B = 1**
= A	**Identity element**

Useful Theorems

- Minimization
  $X Y + X \overline{Y} = Y$
- Absorption
  $X + X Y = X$
- Simplification
  $X + \overline{X} Y = X + Y$

- Minimization (dual)
  $(X+Y)(\overline{X}+Y) = Y$
- Absorption (dual)
  $X \cdot (X + Y) = X$
- Simplification (dual)
  $X \cdot (\overline{X} + Y) = X \cdot Y$

Truth Table to Verify DeMorgan's

$$\overline{X + Y} = \overline{X} \cdot \overline{Y} \qquad \overline{X \cdot Y} = \overline{X} + \overline{Y}$$

X	Y	X·Y	X+Y	$\overline{X}$	$\overline{Y}$	$\overline{X+Y}$	$\overline{X}\cdot\overline{Y}$	$\overline{X \cdot Y}$	$\overline{X}+\overline{Y}$
0	0	0	0	1	1	1	1	1	1
0	1	0	1	1	0	0	0	1	1
1	0	0	1	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0	0

Exercise 1

- Suppose there are three switches x1, x2 and x3 to control a single light bulb L. The bulb is lit only if atleast two of the three switches is ON. Write down the truth table for controlling the bulb.

Exercise 1

Truth Table for Exercise 1

x1	x2	x3	f(x1, x2, x3)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Now write down the function f(x1, x2, x3) for the bulb switching.

To do this we need to understand minterms first

Minterm and Maxterms

- Minterm: For a function of n variables a product terms in which each of the n variables appear once is called a minterm.
- For a given row in the truth table, the minterm is formed by including x_i if x_i =1 and !x_i if !x_i =1.
- Maxterms is the complement of minterm.

Minterms and Maxterms

Minterm	x1	x2	x3	f(x1, x2, x3)	Maxterm
$m0 = \overline{x_1}.\overline{x_2}.\overline{x_3}$	0	0	0	0	$M0 = x_1 + x_2 + x_3$
$m1 = \overline{x_1}.\overline{x_2}.x_3$	0	0	1	0	$M1 = x_1 + x_2 + \overline{x_3}$
$m2 = \overline{x_1}.x_2.\overline{x_3}$	0	1	0	0	$M2 = x_1 + \overline{x_2} + x_3$
$m3 = \overline{x_1}.x_2.x_3$	0	1	1	1	$M3 = x_1 + \overline{x_2} + \overline{x_3}$
$m4 = x_1.\overline{x_2}.\overline{x_3}$	1	0	0	0	$M4 = \overline{x_1} + x_2 + x_3$
$m5 = x_1.\overline{x_2}.x_3$	1	0	1	1	$M5 = \overline{x_1} + x_2 + \overline{x_3}$
$m6 = x_1.x_2.\overline{x_3}$	1	1	0	1	$M6 = \overline{x_1} + \overline{x_2} + x_3$
$m7 = x_1.x_2.x_3$	1	1	1	1	$M7 = \overline{x_1} + \overline{x_2} + \overline{x_3}$

Exercise 1: SOP Expression

$$f(x_1, x_2, x_3) = \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.\overline{x_3} + x_1.x_2.x_3$$

Now, find the minimum Sum-Of-Product (SOP) expression for the function by applying BOOLEAN Algebra.

Exercise 2: Find Minimum SOP

$$f(x1, x2, x3) = x_1.x_3 + x_1.\overline{x_2} + \overline{x_1}.x_2.x_3 + \overline{x_1}.\overline{x_2}.\overline{x_3}$$

Exercise 3: Find Minimum SOP

f(x1, x2, x3) $= \sum m(0, 1, 2, 3, 5)$ ⇨ SOP Form (Look for the ones)

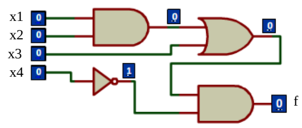
$= \prod M(4, 6, 7)$ ⇨ POS Form (Look for the zeros!!)

Find the minimum Sum-Of-Product (SOP) and Product-Of-Sum (POS) expression for this function by applying BOOLEAN Algebra (**they should be the same if transformed to a common form**!!).

How to Calculate the Cost?

A good indication of the cost of a logic circuit is the total number of gates plus the total number of input to all the gates in the circuit.

$f(x1, x2, x3, x4) = (x1.x2 + x3).\overline{x4}$



Here, No. of
AND Gates = 2
OR Gates = 1
NOT Gates =1
Inputs = (2+2+2+1) = 7

Total Cost = 11

Exercise 3: Cost Calculation

x1	x2	x3	f(x1, x2, x3)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$f(x_1, x_2, x_3) = m_1 + m_4 + m_5 + m_6 = ?$

Cost of f(x1,x2,x3) = ?

$\overline{f(x_1, x_2, x_3)} = m_0 + m_2 + m_3 + m_7 = ?$

$f(x_1, x_2, x_3) = \overline{\overline{f(x_1, x_2, x_3)}}$

$f(x_1, x_2, x_3) = \overline{m_0 + m_2 + m_3 + m_7}$

$= \overline{m_0}.\overline{m_2}.\overline{m_3}.\overline{m_7}$

$= M_0.M_2.M_3.M_7$

Cost of f(x1,x2,x3) = ?

Cost from SOP may not be equal to cost from POS forms.

Exercise 3: Cost Calculation

x1	x2	x3	f(x1, x2, x3)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$f(x_1, x_2, x_3) = m_1 + m_4 + m_5 + m_6 = ?$

Cost of f(x1,x2,x3) = 13 Considering NOT Gate

$\overline{f(x_1, x_2, x_3)} = m_0 + m_2 + m_3 + m_7 = ?$

$f(x_1, x_2, x_3) = \overline{\overline{f(x_1, x_2, x_3)}}$

$f(x_1, x_2, x_3) = \overline{m_0 + m_2 + m_3 + m_7}$

$= \overline{m_0}.\overline{m_2}.\overline{m_3}.\overline{m_7}$

$= M_0.M_2.M_3.M_7$

Cost of f(x1,x2,x3) = 13 Considering NOT Gate

Do not calculate cost for NOT gates if complemented input is available!

That's All for Today!

Try at home:
Chapter 2: Problem 2.9-2.14, 2.19-2.27
(Pages 61-63, 2nd Edition of Stephen Brown's Book)